

in Excuse of this Trouble, farther than that I have
the Honour to be with great Respect,

S I R,

London, Jan. 14.

1746-7.

Your most obliged,

and humble Servant,

Richard Brocklesby.

XIII. *A Letter from Mr. Richard Dunthorne,
to the Rev. Mr. Cha. Mafon, F. R. S. and
Woodwardian Professor of Nat. Hist. at
Cambridge, concerning the Moon's Motion.*

S I R,

Cambridge, Nov. 4. 1746.

Read Feb. 5.
1746 7.

IN the Preface to my lunar Tables, I
hinted, that one Use of publishing
those Tables would be, the assisting of Persons desirous
farther to rectify the lunar Astronomy, by enabling
them more readily to compare the *Newtonian* Theory
with Observations.

Since the Publishing those Tables, I have spent
some Time myself in that Comparison; and here
send you the Result, that you may communicate it
to the *Royal Society*, if you think it deserves to be
made public.

As the Motion of every secondary Planet must
partake of the Errors in the Theory of its primary,
I thought proper, before I undertook the Examina-
tion of the lunar Numbers, to compare those of the
Sun with Observations. I compared several Sets of
Mr.

Mr. *Flamsteed's* Observations, after the Method he himself teaches, in *Prolegom. Hist. Cœlest.* p. 133, & seq. which, for many Reasons, I think the best Method hitherto used; and, with the Concurrence of a Gentleman well skilled in these Matters, determined the mean Motion of the Sun at *Greenwich*, the last Day of *December* at Noon, *Anno 1700, O. S.* ϖ $20^{\circ} 43' 40''$ of its Apogee, \odot $7^{\circ} 30' 0''$, and the greatest Equation of the Sun's Centre $1^{\circ} 55' 40''$; which, I am fully persuaded, are very near the Truth.

The Theory of the Sun being thus settled, I proceeded to examine the Elements of the lunar Astronomy. I began with Observations of lunar Eclipses about the Equinoxes, when the Apogee of the Moon was in the Sun's Quadratures; because at those Times I could conceive the Moon's Motion affected with no Inequality, but the annual one, called by *Newton* the first Equation, and the elliptic one, called *Prosthaphæresis*: From a Comparison of such Observations I obtained the Moon's mean Longitude, which came out $1'$, at least, greater than in the Tables, and very nearly as *Newton* has it in the last Edition of his *Principia*.

I went on to examine the Place and Motion of the Apogee, and Theory of the Increase and Decrease of the Eccentricity, as well as the greatest and least Eccentricities themselves (from the best Observations, and best situate that I could procure) all which agreed so well with the Tables, about the Sun's mean Distances, that I dare venture to make no Alteration therein: Indeed I think the 6th Equation does not so well account for the Variation of the Motion of the Apogee, and Change of the Eccentricity,

tricity, according to the greater or lesser Distance of the Sun from the Earth; and therefore I set myself to compute what Change this Difference of the Sun's Action upon the lunar Orbit would introduce in the Moon's Place in every Situation of the Sun and lunar Orbit; and found, after many tedious Computations, that the Sun being in Apogee, this Change, where greatest, would amount to about $4'$, and to $4' 16''$, when the Sun is in Perigee. In other Distances of the Sun from the Earth, this greatest Change is proportional to the Difference of the Cubes of the mean and present Distances; and in every Situation of the Moon, and of her Orbit, the present is to the greatest Equation nearly as the Sine of the Excess of the Moon's mean Anomaly above twice the annual Argument to *Radius*. It increases the Moon's Longitude, when the Sun is in his $\left\{ \begin{array}{l} \text{Apogee} \\ \text{Perigee} \end{array} \right\}$ Semicircle, and that Excess $\left\{ \begin{array}{l} \text{lets} \\ \text{greater} \end{array} \right\}$ than 180° ; and diminishes it when otherwise *.

In fine, I compared the Theory of the Moon, as to her Longitude, with several Observations, as well in the Octants and Semi-Octants, as in the Syzygies and Quadratures, and found such an Agreement when the above Corrections were made, as seemed rather to be wished than hoped for, considering the many Inequalities wherewith the Sun's Action disturbs the Motion

* If this Equation be increased and diminished in a direct *Ratio* of the Moon's horizontal Parallax, it will become more exact. And I think, if it were always diminished by a fourth or perhaps a third Part, it would agree better with Observations.

Motion of the Moon, and the Defects to which the best Observations I have hitherto met withal are liable.

I have compared 100 observed Longitudes of the Moon with the Tables; *viz.* 25 Eclipses of the Moon, all, except the first, taken from *Flamsteed's Historia Cœlestis*, the *Philosophical Transactions*, and the *Memoirs of the Royal Academy of Sciences*; the two great Eclipses of the Sun in 1706 and 1715; 25 select Places of the Moon from *Flamsteed's Historia Cœlestis*, and 48 of those Longitudes of the Moon computed from *Flamsteed's* Observations by Dr. *Halley* (as I suppose) printed in the first Edition of the *Historia Cœlestis*. They are as follows;

25 Eclipses of the Moon, and 2 of the Sun, compared with the Tables corrected as above.

Place of Ob- servat.	A.D.	Apparent Time at Greenwich.	D's true Place observed.			M. An. om. 2			Ann. Arg't			D's Place comput'ed.			Duration Observat.					
			s	o	''	s	o	''	s	o	''	s	o	''						
Danzick Greenwich Paris	1652	Septem. 7. 6 21 35	11	25	25	51	2	20	6	2	0	3	1	25	26	10	+	0	12	
	1670	Sept. 18. 14 36 48	0	6	13	29	3	1	6	0	0	0	0	0	6	16	3	+	3	8
	1678	Octob. 19. 8 21 54	1	6	47	9	4	1	7	16	1	2	2	1	6	46	33	-	0	31
	1682	Febr. 11. 10 58 52	5	3	47	3	7	25	6	28	1	14	5	3	46	12	-	0	49	
Danzick Dublin Greenwich	1684	Dec. 11. 10 47 6	3	1	11	20	5	24	1	8	7	16	3	1	13	-	+	1	47	
	1685	Nov. 30. 10 35 30	2	19	40	0	5	13	11	26	5	25	2	19	39	46	-	0	14	
	1686	Nov. 19. 11 13 11	2	8	11	57	5	2	10	12	4	8	2	8	11	28	-	0	29	
	1689	Sept. 18. 14 32 37	0	6	32	8	3	1	4	24	10	8	0	6	34	16	+	2	8	
Paris New England Maffelies Greenwich	1690	March 14. 9 36 50	6	4	37	11	8	25	9	18	3	15	6	4	37	28	+	0	17	
	1606	May 6. 12 2 0	7	26	53	14	10	18	3	1	8	28	7	26	53	46	+	0	26	
	1699	March 5. 7 13 44	5	25	32	27	8	16	8	27	3	1	5	25	32	14	-	0	13	
	1703	Decem. 11. 13 29 8	3	0	49	56	5	23	11	24	5	23	3	0	48	51	-	1	5	
Paris London Paris	1706	May 6. 21 13 57	1	20	46	44	10	12	6	22	7	6	1	20	46	32	-	0	12	
	1707	Octob. 10. 7 10 0	0	28	0	20	3	22	5	27	11	25	0	27	58	47	-	1	33	
	1708	April 5. 13 39 0	6	26	18	33	9	17	11	9	5	3	6	26	19	6	+	0	36	
	1708	Sept. 18. 9 10 25	0	6	39	23	3	1	2	12	8	7	0	6	39	10	-	0	4	
	1712	Jan. 12. 7 34 0	4	2	57	47	6	24	1	17	7	25	4	2	58	37	+	0	50	
	1713	Nov. 20. 15 27 30	2	9	58	21	5	3	9	21	3	17	2	9	56	56	-	1	25	
	1715	April 21. 21 11 5	1	12	0	22	10	3	6	12	6	22	1	11	59	36	-	0	46	
	1717	March 15. 15 7 4	6	6	23	50	8	26	8	22	2	27	6	6	23	31	-	0	19	
Paris London Paris	1719	August 18. 8 22 46	1	5	42	8	1	29	10	23	4	20	11	5	42	39	+	0	31	
	1722	June 17. 13 46 10	9	6	47	43	11	29	5	8	10	25	9	6	47	33	-	0	10	
	1724	Octob. 20. 15 40 40	1	9	0	0	4	2	5	25	11	22	1	8	58	50	-	1	10	
	1729	Febr. 2. 8 42 55	4	25	13	39	7	16	3	25	9	14	4	25	14	40	+	1	1	
		July 28. 13 0 0	10	16	15	28	1	9	8	3	2	15	10	16	16	26	+	0	58	
	1731	June 8. 13 47 51	8	28	9	58	11	19	4	27	10	11	8	28	0	48	-	1	13	
1732	Nov. 20. 0 49 25	2	10	3	54	5	3	7	8	1	24	2	10	0	50	-	2	58		

25 Places of the Moon, computed by myself from *Flamsteed's* Observations, compared with the Tables.

A.D.	Apparent Time at Greenwich.	D's true Place observed.			M. An. om. ☉.			M. An. om. ☽.			Ann. Arg ^t .			D's Place computed.			Diff. from Observa ⁿ .					
		s	o	'	s	o	'	s	o	'	s	o	'	s	o	'						
1684	March 13.	8	9	8	2	28	48	40	8	25	2	25	11	19	2	28	50	44	+	2	4	
1693	March 6.	7	22	48	3	16	43	12	8	17	3	11½	11	7	3	16	44	0	+	0	48	
1694	Octob. 11.	18	12	34	3	28	34	2	3	24	2	27	5	15	3	28	38	21	+	4	19	
	Febr. 27.	10	29	16	4	27	27	31	8	10	3	12	9	20	4	27	26	48	—	0	43	
	August 23	11	13	54	11	0	19	11	2	5	8	2	2	2	21	11	0	21	41	+	2	30
1694	Septem. 15.	5	34	13	8	27	1	34	2	27	6	10	3	11	8	27	1	41	+	0	7	
	Septem. 21.	10	50	31	11	22	47	41	3	3	9	3½	3	17	11	22	49	16	+	1	35	
	Decem. 13.	6	2	53	0	6	28	43	5	25	8	26½	6	0	0	6	29	58	+	1	15	
1695	Febr. 8.	3	55	22	1	5	14	3	7	21	9	7½	7	22	1	5	12	14	+	1	49	
	July 9.	5	56	14	7	2	2	42	0	20	3	14	0	2	7	2	3	4	+	0	22	
	Septem. 8.	8	30	26	10	3	0	12	2	20	5	21	1	24	10	2	59	50	—	0	22	
1696	January 16.	17	29	2	7	5	4	20	6	29	2	29	5	20½	7	5	6	11	+	1	51	
	March 4.	9	8	13	4	12	2	24	8	16	11	8	7	3	4	12	0	59	—	1	25	
1697	February 18.	6	29	47	2	20	52	29	8	2	8	19	5	10	2	20	51	7	—	1	22	
	19.	7	22	57	3	4	18	16	8	3	9	2½	5	11	3	4	17	54	—	0	22	
	Septem. 15.	7	54	4	10	1	7	8	2	23	3	21½	11	9	10	1	5	38	—	1	30	
1698	Septem. 8.	11	2	35	11	12	15	6	2	21	3	24	9	23	11	12	15	27	+	0	21	
	Novem. 27.	3	49	54	10	11	33	39	5	9	2	2½	0	3	10	11	31	13	+	2	26	
1699	March 5.	12	8	23	5	28	15	59	8	16	8	29½	3	1	5	28	16	11	+	0	12	
1701	Septem. 28.	6	55	23	9	27	13	17	3	10	9	9	6	8	9	27	12	49	—	0	28	
1702	Octob. 16.	6	15	49	10	3	25	41	3	28	8	11½	5	13	10	3	26	27	+	0	46	
1703	Septem. 13.	11	58	50	11	28	50	30	2	25	8	28	3	3	11	28	53	30	+	3	0	
	Octob. 6.	6	28	39	9	27	1	44	3	18	7	4	3	23	9	27	0	38	—	1	6	
1706	Octob. 10.	11	11	10	1	11	15	13	3	22	6	0	11	25	1	10	17	—	—	0	58	
1714	Septem. 6.	6	34	7	9	3	44	13	2	18	3	17	0	0	9	3	45	24	+	1	11	

48 Places of the Moon, computed by Dr. Halley, from *Flamsteed's* Observations, compared with the Tables.

A.D.	Apparent Time at Greenwich.	D's true Place observed.			M. An. om. O.	M. An. om. D.			Ann. Arg.	D's Place computed.			Diff. from Observat.			
		h	'	"		s	o	'		s	o	'		s	o	'
1689	Novem. 16. 11 59 0	2	6	12 43	4	29	6	1	0	2	6	14 40	+	1	57	
	Decem. 9. 6 1 0	11	28	43 54	5	21½	3	21	0	11	28	48 40	+	0	14	
	10. 6 46 35	0	12	46 49	5	22½	4	4	0	0	12	47 14	+	0	25	
	12. 8 26 33	1	12	13 8	5	24½	5	0½	0	23	1	12	10 35	+	2	29
	13. 9 24 30	1	27	38 36	5	25½	5	14	0	24	1	27	36 16	—	2	20
1690	16. 12 42 0	3	15	14 54	5	28½	6	24	0	27	3	15	15 48	+	0	54
	January 4. 3 3 46	11	11	12 59	6	17	2	24	1	14	11	11 13 8	+	0	9	
	6. 4 30 15	0	8	13 31	6	19	3	20½	1	16	0	8 14 36	+	1	5	
	10. 7 59 22	2	6	11 20	6	23	5	14½	1	19½	2	6 10 37	+	0	43	
	12. 10 8 49	3	7	5 21	6	25	6	121	1	21	3	7 3 59	—	1	22	
February	13. 11 14 0	3	22	36 28	6	26	6	25½	1	22	3	22 35 19	—	1	9	
	2. 2 25 39	0	3	57 24	7	15½	3	5	2	10	0	3 56 9	—	1	15	
	5. 4 51 10	1	16	31 33	7	18½	4	26	2	12½	1	16 34 15	+	2	42	
	7. 6 48 17	2	15	58 14	7	20½	5	24	2	14½	2	15 59 13	+	0	59	
	8. 7 51 54	3	0	56 20	7	21½	6	8	2	15½	3	0 55 31	+	0	11	
	10. 9 56 26	4	0	55 24	7	24	7	6	2	17	4	0 55 1	—	0	23	
	11. 10 52 31	4	15	42 16	7	25	7	20	2	18	4	15 41 49	—	0	27	
	14. 13 19 31	5	28	12 40	7	28	9	2	2	21	5	28 11 44	—	0	56	
	19. 17 3 55	8	2	23 20	8	3	11	12	2	25½	8	2 21 17	—	2	9	
	21. 18 45 37	8	27	1 25	8	5	0	10	2	27½	8	26 59 6	—	2	19	

1690	Febr.	22. 19 37 46	9	9 21 32	8	6	0 24	2 28	9 9 23 26	+	1 54
	March	7. 5 50 38	2	26 22 50	8 18	6	6 12	3 9	2 26 25 20	+	2 30
		11. 9 43 23	4	24 31 45	8 22	8	8 8	3 13	4 24 33 15	+	1 30
		12. 10 33 9	5	8 31 54	8 23	8	8 22	3 14	5 8 33 11	+	1 17
		13. 11 18 30	5	22 16 37	8 24	9	9 5½	3 15	5 22 17 26	+	0 49
		14. 12 2 18	6	5 44 39	8 25	9	9 16½	3 15½	6 5 45 18	+	0 39
		25. 20 54 56	10	24 54 20	9 6½	2	2 21	3 25½	10 24 53 20	+	1 18
	April	7. 7 48 46	4	20 8 0	9 19	8	8 5	4 6	4 20 9 18	+	0 54
		8. 8 36 30	5	3 59 40	9 20	8	8 19	4 7	5 4 0 34	+	0 58
		9. 9 22 0	5	17 34 42	9 21	9	9 2	4 8	5 17 35 40	+	0 34
		10. 10 5 19	6	0 54 50	9 22	9	9 16	4 9	6 0 55 35	+	0 45
		11. 10 47 56	6	14 0 8	9 23	9	9 29	4 10	6 14 2 13	+	2 5
		12. 11 30 56	6	26 55 56	9 24	10	10 12½	4 11	6 26 56 55	+	0 59
		13. 12 17 31	7	9 41 19	9 25	10	10 26	4 12	7 9 42 20	+	1 1
		14. 13 4 0	7	22 16 0	9 26	11	11 9½	4 12½	7 22 17 34	+	1 34
		15. 13 52 44	8	4 45 10	9 27	11	11 23	4 13½	8 4 44 47	+	0 23
		18. 16 26 47	9	11 43 52	10 0	1	1 3½	4 16	9 11 44 53	+	1 1
		20. 18 5 47	10	6 30 55	10 2	2	2 0	4 18	10 6 33 2	+	2 7
		22. 19 37 30	11	2 8 6	10 4	2	2 27	4 20	11 2 8 45	+	0 39
		24. 21 6 38	11	29 14 33	10 6	3	3 23½	4 21½	11 29 14 30	+	0 3
	May	14. 13 30 30	8	25 6 53	10 25½	0	0 7	5 8	8 25 8 48	+	1 55
		22. 19 41 40	0	6 50 26	11 3½	3	3 23	5 15	0 6 49 43	+	0 43
	June	3. 6 2 13	5	22 31 45	11 15	8	8 19	5 25	5 22 32 6	+	0 21
		1. 6 48 53	9	2 27 42	2 13½	11	11 15	8 12	9 2 31 20	+	3 38
	Septem.	8. 12 16 42	0	1 44 51	2 21	2	2 22	8 18	0 1 44 18	+	0 32
1691	February	23. 3 55 10	1	19 39 56	8 6	3	3 16	1 18	1 19 40 45	+	0 49
	March	14. 18 27 51	9	7 30 5	8 22½	10	10 23½	2 2	9 7 30 23	+	0 18
	Septem.	22. 8 51 31	10	19 26 24	3 4	11	11 16	7 19	10 19 28 32	+	2 8

a, The Time of the Middle of this Eclipse here set down is from the Beginning and End ; but *Hevelius* says he could not observe the Beginning exactly. Several intermediate *Phases* compared together shew the Middle to have been about 4' sooner ; to which the Moon's Place computed is $os. 6^{\circ}. 14'. 3''.$ and Diff. $+ 34''.$

b, b, b, The Moon's Places, observed on *Feb. 2. April 7. and May 22.* are computed by myself, from the Observations ; there being manifestly Errors, either of the Computation or Press, in those printed in the *Hist. Cælestis*.

Several observed Latitudes of the Moon, which I have compared with the Tables, shew them to be very near the Truth, both in the Motion of the Nodes, and also in the Quantity and Variation of the Inclination. I am,

S I R,

Your humble Servant,

Richard Dunthorne.